

# Overview of the parametric representation of renormalizable non-commutative field theory

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**Abstract.** We review here the parametric representation of Feynman amplitudes of renormalizable non-commutative quantum field models.

## I. Introduction

In the quest for quantification of gravity, non-commutative geometry (see [1]) is nowadays one of the most appealing concepts. Non-commutative quantum field theory (NCQFT) - for some general reviews see [2] or [3] - is also an interesting candidate for New Physics beyond the Standard Model. Moreover, let us also note that NCQFT can be seen as an effective limit of string theory [4, 5]).

In a completely different framework, non-commutativity can be important for the understanding of physics in the presence of a background field (like is the case for example for the fractional quantum Hall effect [6, 7, 8]).

Because of a new type of divergence - the UV/IR mixing - NCQFTs were thought to be non-renormalizable. The situation changed for scalar fields with the Grosse-Wulkenhaar model, model which modifies the kinetic part of the action. This modification takes into consideration a symmetry already present at the level of the interaction term, the Langman-Szabo duality [9], duality between the IR and UV regions). This model was proven, by different methods, to be renormalizable to any order in perturbation theory [10, 11, 12, 13, 14]. The parametric representation for it was then implemented [15]. This was the starting point for the proof of dimensional renormalization [16]. Recently, this modification of the kinetic part of the action was proven [17] to be also related to the spectral action principle (for last advances to this issue see [18]).

The Grosse-Wulkenhaar model was moreover proven to have a better theoretical behavior with respect to the commutative  $\phi^4$  model. Indeed, in [19, 20] and [21] was proven that this model does not present a Landau ghost; let us recall that this was not the case for the commutative model.

Another improvement with respect to commutative scalar quantum field theory is that a constructive version (for a general review see [22]) is within reach [23, 24].

An analyze of the vacuum states of the model as well as an investigation of spontaneous symmetry breaking and the Goldstone theorem for a corresponding linear sigma model were also realized [25].

Moreover, a second type of renormalizable NCQFT models - the *covariant* models - exists [26]. This classification is done with respect to the form of the propagator in position space. This second category of NCQFTs contains the non-commutative Gross-Neveu model and the Langmann-Szabo-Zarembo (LSZ) model [27]. The parametric representation for the latter was also obtained [28]. One can report himself to [29] or [30] for some general review of renormalizable NCQFTs.

For both types of models the Mellin representation of the Feynman amplitudes was implemented [31]. Furthermore, a Hopf algebra structure associated to this new type of renormalization was defined [32].

Finally, let us also state that recent progress has been obtained in the search of renormalizable non-commutative gauge theories [17, 33, 34, 35, 36, 37].

We present here an overview of the parametric representation for both known types of renormalizable NCQFTs, overview based on [15] and [28]. The paper is structured as follows. In the next section, a brief recall of the parametric representation for commutative QFT is given. In the third section we introduce the two types of models we consider here: the Grosse-Wulkenhaar model and the covariant models. The next section presents our main results, namely the implementation of the parametric representation for both these models. Finally, in the last section we give a few examples for some simple Feynman graphs.

## II. Parametric representation for commutative QFT

The parametric representation relies on the introduction of an integral representation on some Schwinger parameters  $\alpha_\ell$  ( $\ell = 1, \dots, L$ ,  $L$  being the total number of internal lines of the graph). One has

$$\frac{1}{p_\ell^2} = \int_0^\infty d\alpha_\ell e^{-\alpha_\ell k_\ell^2}.$$

The amplitude of a Feynman graph for a non-massif model writes (see for example [22] or [38] for further details)

$$\mathcal{A}(p) = \delta(\sum p) \int_0^\infty \frac{e^{-V(p,\alpha)/U(\alpha)}}{U(\alpha)^2} \prod_{\ell=1}^L d\alpha_\ell. \quad (\text{II.1})$$

The  $U$  and  $V$ , so called “topological” or “Symanzik” polynomials, are proven to be polynomials in the Schwinger parameters  $\alpha$  and they have for the  $\phi^4$  model the expressions

$$U = \sum_{\mathcal{T}} \prod_{l \notin \mathcal{T}} \alpha_l, \quad (\text{II.2})$$

$$V = \sum_{\mathcal{T}_2} \prod_{l \notin \mathcal{T}_2} \alpha_l \left( \sum_{i \in E(\mathcal{T}_2)} p_i \right)^2, \quad (\text{II.3})$$

where  $\mathcal{T}$  is a (spanning) tree of the graph and  $\mathcal{T}_2$  is a 2-tree, *i. e.* a tree minus one of its lines. Any such 2-tree splits the graph into exactly two connected components which we denote  $E(\mathcal{T}_2)$  and  $F(\mathcal{T}_2)$ . Moreover the sum  $\sum_{i \in E(\mathcal{T}_2)} p_i$  (where  $p_i$  is an external momentum) is, by momentum conservation, also equal to  $(\sum_{i \in F(\mathcal{T}_2)} p_i)^2$ .

### III. Renormalizable NCQFT models

We place ourselves in a 4-dimensional Moyal space

$$[x^\mu, x^\nu] = i\Theta^{\mu\nu}, \quad (\text{III.1})$$

where the matrix  $\Theta$  is

$$\Theta = \begin{pmatrix} 0 & \theta & 0 & 0 \\ -\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta \\ 0 & 0 & -\theta & 0 \end{pmatrix}. \quad (\text{III.2})$$

The associative Moyal product of two functions  $f$  and  $g$  on the Moyal space writes

$$(f \star g)(x) = \int \frac{d^4 k}{(2\pi)^4} d^4 y f(x + \tfrac{1}{2}\Theta \cdot k) g(x + y) e^{ik \cdot y} \quad (\text{III.3})$$

We also consider Euclidean metric. Let us now introduce in the rest of this section the two types of renormalizable non-commutative models.

#### III.1. The Grosse-Wulkenhaar model

Note that the results established in the sequel hold for orientable models (that is interactions  $\bar{\phi} \star \phi \star \bar{\phi} \star \phi$ ). This corresponds to a Grosse-Wulkenhaar model of a complex scalar field

$$S_{GW} = \int d^4 x \left( \frac{1}{2} \partial_\mu \bar{\phi} \star \partial^\mu \phi + \frac{\Omega^2}{2} (\tilde{x}_\mu \bar{\phi}) \star (\tilde{x}^\mu \phi) + \bar{\phi} \star \phi \star \bar{\phi} \star \phi \right) \quad (\text{III.4})$$

where

$$\tilde{x}_\mu = 2(\Theta^{-1})_{\mu\nu} x^\nu \quad (\text{III.5})$$

This action leads to the following propagator from a point  $x$  to a point  $y$ :

$$C(x, y) = \int_0^\infty \frac{\tilde{\Omega} d\alpha}{[2\pi \sinh(\alpha)]^2} e^{-\frac{\tilde{\Omega}}{4} \coth(\frac{\alpha}{2})(x-y)^2 - \frac{\tilde{\Omega}}{4} \tanh(\frac{\alpha}{2})(x+y)^2}. \quad (\text{III.6})$$

Let us now introduce the *short* and *long variables*:

$$u = \frac{1}{\sqrt{2}}(x - y), \quad v = \frac{1}{\sqrt{2}}(x + y). \quad (\text{III.7})$$

and

$$t_\ell = \tanh \frac{\alpha}{2}. \quad (\text{III.8})$$

The propagator (III.6) becomes

$$C(x, y) = \int_0^\infty \frac{\tilde{\Omega} d\alpha}{[2\pi \sinh(\alpha)]^2} e^{-\frac{\tilde{\Omega}}{2} \frac{1}{t_\ell} u^2 - \frac{\tilde{\Omega}}{2} t_\ell v^2}. \quad (\text{III.9})$$

### III.2. The covariant models

Amongst this type of models one has, as already stated in the introduction, the non-commutative Gross-Neveu model and the LSZ model. The results we establish in the sequel hold for the latter but they can be however extended for the Gross-Neveu model also. The LSZ action writes:

$$S_{LSZ} = \int d^4x \left( \frac{1}{2} (\partial_\mu \bar{\phi} - i\Omega \tilde{x}_\mu \phi) \star (\partial^\mu \phi - i\Omega \tilde{x}^\mu \phi) + \bar{\phi} \star \phi \star \bar{\phi} \star \phi \right). \quad (\text{III.10})$$

This action leads to the propagator

$$C(x, y) = 2 \int_0^1 dt_\ell \frac{\tilde{\Omega}(1 - t_\ell^2)}{(4\pi t_\ell)^2} e^{-\frac{1}{2}\tilde{\Omega} \frac{1+t_\ell^2}{2t_\ell} u^2 + i\tilde{\Omega} u \wedge v}, \quad (\text{III.11})$$

where

$$u \wedge v = u_1 v_2 - u_2 v_1 + u_3 v_4 - u_4 v_3.$$

### III.3. Non-local interaction and hypermomenta

Using the form (III.3) of the Moyal product, the interaction term of both (III.4) and (III.10) lead to the following contribution in position space

$$\delta(x_1^V - x_2^V + x_3^V - x_4^V) e^{2i \sum_{1 \leq i < j \leq 4} (-1)^{i+j+1} x_i^V \Theta^{-1} x_j^V} \quad (\text{III.12})$$

where  $x_1^V, \dots, x_4^V$  are the 4-vectors of the positions of the 4 fields incident to the respective vertex.

To any such vertex one associates a hypermomentum  $p_V$  via the relation

$$\delta(x_1^V - x_2^V + x_3^V - x_4^V) = \int \frac{dp_V}{(2\pi)^4} e^{p_V \sigma (x_1^V - x_2^V + x_3^V - x_4^V)} \quad (\text{III.13})$$

## IV. Parametric representation

In the case of commutative QFT, one has translation invariance in position space. As a consequence of this invariance, the first polynomial  $U$  (see section II) vanishes when integrating over all internal positions. Therefore, one has to integrate over all internal positions (which correspond to vertices) save one, which is thus singularized. However, the polynomial is a still a canonical object, *i. e.* it does not depend of the choice of this particular vertex.

As noticed in [15], in the non-commutative case the translation invariance is lost. Therefore, one can integrate over all internal positions and hypermomenta, without vanishing of the first polynomial. However, in order to be able to recover the commutative limit, we also singularize a particular vertex  $\bar{V}$ ; we do not integrate on its associate hypermomenta  $p_{\bar{V}}$ . We call this particular vertex the *root*. Because there is no translation invariance, the polynomial does depend on the choice of the root; however the leading ultraviolet terms do not.

We now define the  $(L \times 4)$ -dimensional incidence matrix  $e^V$  for each of the vertices  $V$ . Since the graph is orientable (in the sense defined in the previous section) we can choose

$$\epsilon_{\ell i}^V = (-1)^{i+1}, \text{ if the line } \ell \text{ hooks to the vertex } V \text{ at corner } i. \quad (\text{IV.1})$$

We also put

$$\eta_{\ell, i}^V = |\epsilon_{\ell, i}^V|, \quad V = 1, \dots, n, \quad \ell = 1, \dots, L \text{ and } i = 1, \dots, 4. \quad (\text{IV.2})$$

From (IV.1) and (IV.2) one has

$$\eta_{\ell i}^V = (-1)^{i+1} \epsilon_{\ell i}. \quad (\text{IV.3})$$

We now generalize the short and long variables introduced in the previous section at the level of the whole Feynman graph:

$$\begin{aligned} v_\ell &= \frac{1}{\sqrt{2}} \sum_V \sum_i \eta_{\ell i}^V x_i^V, \\ u_\ell &= \frac{1}{\sqrt{2}} \sum_V \sum_i \epsilon_{\ell i}^V x_i^V. \end{aligned} \quad (\text{IV.4})$$

Conversely, one has

$$x_i^V = \frac{1}{\sqrt{2}} (\eta_{\ell i}^V v_\ell + \epsilon_{\ell i}^V u_\ell).$$

From the propagator (III.9) and the vertices contributions (III.12) one is able to write the amplitude  $\mathcal{A}_{G, \bar{V}}$  of the graph  $G$  (with the singularized root  $\bar{V}$ ) as function of the non-commutative polynomials  $HU_{G, \bar{V}}$  and  $HV_{G, \bar{V}}$  as (see [15] for details)

$$\mathcal{A}_{G, \bar{V}}(x_e, p_{\bar{V}}) = K \int_0^1 \prod_{\ell=1}^L [dt_\ell (1 - t_\ell^2)] HU_{G, \bar{V}}(t)^{-2} e^{-\frac{HV_{G, \bar{V}}(t_\ell, x_e, p_{\bar{V}})}{HU_{G, \bar{V}}(t)}}, \quad (\text{IV.5})$$

where  $K$  is some constant, unessential for this calculus and by  $x_e$  we mean the external positions of the graph, In [15] it was furthermore proved that  $HU$  and  $HV$  are polynomials in the set of variables  $t$ .

Let us state that, even the formulas above hold also for non-orientable graphs (that is graphs corresponding to interactions  $\bar{\phi} \star \bar{\phi} \star \phi \star \phi$ ), for simplicity reasons we restrict ourselves to the study of polynomials for orientable graphs (that is graphs corresponding to interactions  $\bar{\phi} \star \phi \star \bar{\phi} \star \phi$ , as already mentioned in the previous section). One has (see again [15])

$$HU_{G, \bar{V}} = (\det Q)^{\frac{1}{4}} \prod_{\ell=1}^L t_\ell \quad (\text{IV.6})$$

where

$$Q = A \otimes 1_D - B \otimes \sigma, \quad (\text{IV.7})$$

with  $A$  a diagonal matrix and  $B$  an antisymmetric matrix.

#### IV.1. The parametric representation for the Grosse-Wulkenhaar model

The matrix  $A$  writes

$$A = \begin{pmatrix} S & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (\text{IV.8})$$

where  $S$  and resp.  $T$  are the two diagonal  $L$  by  $L$  matrices with diagonal elements  $c_\ell = \coth(\frac{\alpha_\ell}{2}) = 1/t_\ell$ , and resp.  $t_\ell$ . The last  $(n-1)$  lines and columns are have 0 entries.

The antisymmetric part  $B$  is

$$B = \begin{pmatrix} sE & C \\ -C^t & 0 \end{pmatrix} \quad (\text{IV.9})$$

with

$$s = \frac{2}{\theta\tilde{\Omega}} = \frac{1}{\Omega}$$

and

$$C_{\ell V} = \left( \frac{\sum_{i=1}^4 (-1)^{i+1} \epsilon_{\ell i}^V}{\sum_{i=1}^4 (-1)^{i+1} \eta_{\ell i}^V} \right), \quad (\text{IV.10})$$

$$E = \begin{pmatrix} E^{uu} & E^{uv} \\ E^{vu} & E^{vv} \end{pmatrix}, \quad (\text{IV.11})$$

where

$$\begin{aligned} E_{\ell, \ell'}^{vv} &= \sum_V \sum_{i,j=1}^4 (-1)^{i+j+1} \omega(i, j) \eta_{\ell i}^V \eta_{\ell' j}^V, \\ E_{\ell, \ell'}^{uu} &= \sum_V \sum_{i,j=1}^4 (-1)^{i+j+1} \omega(i, j) \epsilon_{\ell i}^V \epsilon_{\ell' j}^V, \\ E_{\ell, \ell'}^{uv} &= \sum_V \sum_{i,j=1}^4 (-1)^{i+j+1} \omega(i, j) \epsilon_{\ell i}^V \eta_{\ell' j}^V. \end{aligned} \quad (\text{IV.12})$$

Note that  $\omega$  is the antisymmetric matrix for whom  $\omega(i, j) = 1$  if  $i < j$ . Finally, in order to have the integer expression (IV.10) of the matrix  $C$  we have rescaled by  $s$  the hypermomenta  $p_V$ . We also define the integer entries matrix:

$$B' = \begin{pmatrix} E & C \\ -C^t & 0 \end{pmatrix}. \quad (\text{IV.13})$$

In [15] it was also proven that

$$\det Q = (\det M)^4 \quad (\text{IV.14})$$

where

$$M = A + B \quad (\text{IV.15})$$

and thus (IV.6) becomes:

$$HU_{G, \bar{V}} = \det M \prod_{\ell=1}^L t_{\ell} \quad (\text{IV.16})$$

Let now  $I$  and resp.  $J$  be two subsets of  $\{1, \dots, L\}$ , of cardinal  $|I|$  and resp.  $|J|$ . Moreover, let

$$k_{I,J} = |I| + |J| - L - F + 1 \quad (\text{IV.17})$$

and  $n_{IJ} = \text{Pf}(B'_{\hat{I}\hat{J}})$ , the Pfaffian of the matrix  $B'$  with deleted lines and columns  $I$  among the first  $L$  indices (corresponding to short variables  $u$ ) and  $J$  among the next  $L$  indices (corresponding to long variables  $v$ ).

The specific form (IV.8) allows one to write the polynomial  $HU$  as a sum of positive terms:

$$HU_{G,\bar{V}}(t) = \sum_{I,J} s^{2g-k_{I,J}} n_{I,J}^2 \prod_{\ell \notin I} t_\ell \prod_{\ell' \in J} t_{\ell'} . \quad (\text{IV.18})$$

where  $g$  is the genus of the graph.

In [15], non-zero *leading terms* (*i. e.* terms which have the smallest global degree in the  $t$  variables) were identified. These terms are dominant in the UV regime. Some of them correspond to subsets  $I = \{1, \dots, L\}$  and  $J$  admissible, where by *admissible* we mean that

- it contains a tree  $\mathcal{T}$  in the dual graph,
- its complement contains a tree  $\mathcal{T}$  in the direct graph.

In this case,

$$n_{I,J} = 2^{2g-k_J}$$

where  $k_J$  is nothing but  $k_{I,J}$  given by (IV.17) with  $|I| = L$ . This allows us to set a lower limit on the polynomial  $HU$ :

$$HU_{G,\bar{V}} \geq \sum_{J \text{ admissible}} (2s)^{2g-k_J} \prod_{\ell \in J} t_\ell, \quad (\text{IV.19})$$

This is the main result obtained in [15].

#### IV.2. Parametric representation for the Langman-Szabo-Zarembo Model

As stated above, the different form of the propagator (III.11) (with respect to the Grosse-Wulkenhaar propagator (III.9)) leads to a series of changes in the parametric representation for the covariant models, changes which we now list (see [28] for further details). The diagonal matrix  $A$  writes

$$A = \begin{pmatrix} S & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (\text{IV.20})$$

where  $S$  is a  $L$ -dimensional diagonal matrix with elements  $\frac{1+t_\ell^2}{2t_\ell}$ .

The antisymmetric part  $B$  has the same form as (IV.9), the only difference being in:

$$E_{\ell,\ell'}^{uv} = \sum_V \sum_{i,j=1}^4 (-1)^{i+j+1} \omega(i,j) \epsilon_{\ell i}^V \eta_{\ell' j}^V + 2\Omega \delta_{\ell\ell'}. \quad (\text{IV.21})$$

Let now  $K$  be a subset of  $\{1, \dots, L\}$ . Let

$$k_K = |K| - L - F + 1 \quad (\text{IV.22})$$

and  $n_K = \text{Pf}(B'_K)$ , the Pfaffian of the matrix  $B'$  with deleted lines and columns  $K$  among the first  $L$  indices (corresponding to short variables  $u$ ).

The specific form (IV.20) allows, as in the previous subsection, to write the polynomial  $HU$  as a sum of positive terms:

$$HU_{G,\bar{V}}(t) = \sum_K s^{2g-k_K} n_K^2 \prod_{\ell \in K} \frac{1+t_\ell^2}{2t_\ell} \prod_{\ell' \in \{1,\dots,L\}} t_{\ell'}. \quad (\text{IV.23})$$

As for the Grosse-Wulkenhaar model, it was proven in [28] that one can compute some leading terms. Indeed, when choosing

$$K = \{1, \dots, L\} - J_0,$$

where  $J_0$  is an admissible set (as defined in the previous subsection), one has

$$n_K = 2^g \prod 2(\Omega \pm 1)$$

(the product of the factors  $(\Omega \pm 1)$  depending on the topology of the graph). As in the previous section one can now set a lower limit on the polynomial  $HU$

$$HU_{G,\bar{V}}(t) \geq \sum_{J_0 \text{ admissible}} s^{2[g+(F-1)]} \left( 2^g \prod 2(\Omega \pm 1) \right)^2 \prod_{\ell \in I} \frac{1+t_\ell^2}{2t_\ell} \prod_{\ell' \in \{1, \dots, L\}} t_{\ell'}, \quad (\text{IV.24})$$

and this is the main result of [28].

The main results (IV.19) and (IV.24) allow one to obtain the following power counting for both these models

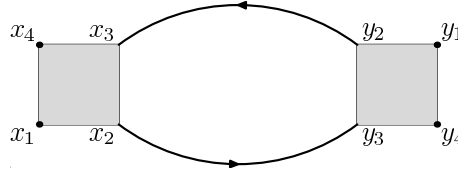
$$\omega = 4g + \frac{1}{2}(N - 4),$$

where  $\omega$  is the superficial divergence degree and  $N$  is the number of external legs of the respective Feynman graph.

Note that for both type of models, similar results of positivity, boundness and power counting have been obtained in [15] and resp. [28] for the second polynomial  $HV$  too.

## V. Examples

In this section we give some examples of the polynomials  $HU$ . Note that the root is always chosen to be the vertex denoted by  $(x_1, \dots, x_4)$ .



**Figure 1.** The bubble graph

For the bubble graph (see Fig. 1) one has, in the case of the Grosse-Wulkenhaar model,

$$HU_{G,\bar{V}} = (1 + 4s^2)(t_1 + t_2 + t_1^2 t_2 + t_1 t_2^2). \quad (\text{V.1})$$

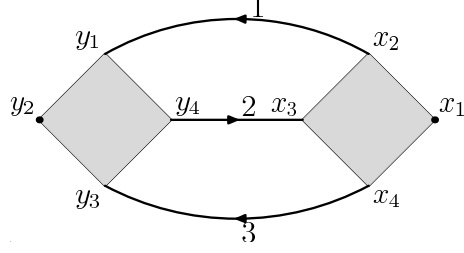
In the case of the LSZ model, one has

$$HU_{G,\bar{V}} = 2s^2(t_1 + t_2 + t_1^2 t_2 + t_1 t_2^2)(\Omega - 1)^2. \quad (\text{V.2})$$

For the sunshine graph (see Fig. 2) one has, in the case of the Grosse-Wulkenhaar model,

$$HU_{G,\bar{V}} = \left[ t_1 t_2 + t_1 t_3 + t_2 t_3 + t_1^2 t_2 t_3 + t_1 t_2^2 t_3 + t_1 t_2 t_3^2 \right] (1 + 8s^2 + 16s^4) + 16s^2(t_2^2 + t_1^2 t_3^2). \quad (\text{V.3})$$



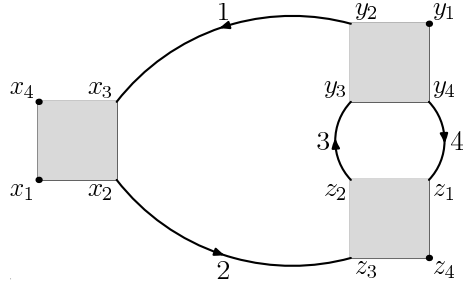


**Figure 2.** The sunshine graph

In the case of the LSZ model, one has:

$$HU_{G,\bar{V}} = 8s^4(t_1t_2 + t_1t_3 + t_2t_3 + t_1^2t_2t_3 + t_1t_2^2t_3 + t_1t_2t_3^2)(\Omega - 1)^2(\Omega + 1)^2. \quad (\text{V.4})$$

We end this section by a more complicated example.



**Figure 3.** The half-eye Graph

For the Grosse-Wulkenhaar model, the graph of Fig. 3 leads to

$$\begin{aligned} HU_{G,v_1} = & (A_{24} + A_{14} + A_{23} + A_{13} + A_{12})(1 + 8s^2 + 16s^4) \\ & + t_1t_2t_3t_4(8 + 16s^2 + 256s^4) + 4t_1t_2t_3^2 + 4t_1t_2t_4^2 \\ & + 16s^2(t_3^2 + t_2^2t_4^2 + t_1^2t_4^2 + t_1^2t_2^2t_3^2) \\ & + 64s^4(t_1t_2t_3^2 + t_1t_2t_4^2), \end{aligned} \quad (\text{V.5})$$

(where for example  $A_{24} = t_1t_3 + t_1t_3t_2^2 + t_1t_3t_4^2 + t_1t_3t_2^2t_4^2$ ). For the LSZ model one has

$$\begin{aligned} HU_{G,\bar{V}} = & 4s^4(\Omega - 1)^2(\Omega + 1)^2[t_3t_4 + t_2^2t_3t_4 + t_2(t_3 + t_4 + t_3^2t_4 + t_3t_4^2) \\ & + t_1^2(t_3t_4 + t_2^2t_3t_4 + t_2(t_3 + t_4 + t_3^2t_4 + t_3t_4^2)) \\ & + t_1((1 + t_2^2)(t_4 + t_3^2t_4) + t_3(1 + 64s^2t_2t_4 + t_4^2 + t_2^2(1 + t_4^2)))]. \end{aligned} \quad (\text{V.6})$$

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